Vector Fitting based Adaptive Frequency Sampling for Compact Model Extraction on HPC Systems

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***Abstract*—This paper proposes an accurate and robust parallel algorithm for the extraction of reduced order electromagnetic models for passive components, by taking into consideration high frequency field effects. It aims at an extraction time as low as possible while keeping the accuracy control of the extracted model. The core of the reduction procedure is based on Vector Fitting rational approximation of the circuit functions, which are computed by solving the field equations with the Finite Integration Technique in a minimal list of frequency samples. The computationally intensive part of algorithm is efficiently mapped on a non-homogeneous hierarchical multiprocessor architecture.**

I. Motivation

Electromagnetic (EM) field models that take into account skin effect, dielectric losses, wave propagation are required in the design of several advanced devices, e.g. high frequency integrated circuits (HF-ICs). They lead to models that may have over one million degrees of freedom even in the case of simple passive components such as spiral inductors, meander resistors, on-chip capacitors or metallic interconnects. Since the simulation of the complete blocks of HF-ICs is done by aggregating their constitutive parts, a model order reduction (MOR) technique, which transform these huge models to much smaller ones, more or less equivalent, but easy to simulate is required.

In our previous experience, we have successfully used Vector Fitting (VF) as a MOR procedure for passive components. The VF algorithm starts from the values of the transfer function in a set of given frequency samples and finds the best rational approximation of this frequency characteristic [1]. The accuracy of the models extracted by VF depends on the number and distribution of frequency samples. For the problems we consider, there is no prior knowledge of the frequency characteristic. That is why a high accuracy would need a large number of samples to be used. However, each sample is costly because it needs a full-wave EM simulation. So, without an accuracy control the computation time may increase uncontrollably in the case of over-sampling or the extracted model may be poor in the case of local under-sampling. In order to reduce the number of samples that need to be evaluated, various adaptive sampling techniques (AFS) techniques have been proposed. In [2] rational macromodels of EM systems with a reduced cost have been generated by using an AFS algorithm combined with VF, procedure that proved to be much stable than traditional least squares approaches. Multiple rational models are built, ranked and the best two are retained, the fitting error being based on the difference between these two models. The AFS technique we proposed in [3] uses a different fitting error, the accuracy of the reduced model being estimated in a set of ”test frequencies” interleaved among ”sample frequencies”. By controlling the accuracy in the intervals between samples, the total number of frequencies is kept at the lowest level, thus reducing the model extraction effort. In this paper we focus on the implementation and testing of this algorithm on an non-homogeneous multi-CPU architecture.

II. Problem Formulation and Algorithm

The first modeling step is to chose an appropriate EM field formulation and thus define a consistent input/output system which has a well defined response, described by the unique output signals, for any input signals applied as terminal excitations. The most appropriate formulation for our class of problems is the Electro-Magnetic Circuit Element (EMCE) [4]. The next modeling step is done by applying the Finite Integration Technique (FIT) to discretize the continuous model defined above. Subsequently, a semi-state space model can be obtained:

$$\mathbf{C}\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = -\mathbf{G}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad \mathbf{y}(t) = \mathbf{L}\mathbf{x}(t) \qquad (1)$$

where $\mathbf{x}$ is the state vector, $\mathbf{u}$ and $\mathbf{y}$ are the vectors of input and output quantities, respectively. The relationship between the complex representations of the input/output signals is given in the frequency domain by the transfer matrix

$$\mathbf{H}_{\mathrm{FIT}}(\omega) = \mathbf{L}(\mathbf{G} + j\omega\mathbf{C})^{-1}\mathbf{B}. \qquad (2)$$

Thus, to compute (2), an algebraic system of linear complex equations has to be solved for each frequency. The size of system depends on the fineness of the FIT discretization grid (which may contain a huge number of nodes), and on the number of terminals which gives the number of right hand side terms.

The VF order reduction procedure uses as input a set of values $(\omega_k, \mathbf{H}(\omega_k))$, $k = 1, F$, where F is the number of frequency samples and it identifies the poles p_i (scalars, real or complex conjugate pairs), the residual matrices $\mathbf{K}_i$ and the constant terms $\mathbf{K}_\infty$, $\mathbf{K}_0$ of the rational approximation

$$\mathbf{H}_{\mathrm{VF}}(\omega) = \sum_{i=1}^{q} \frac{\mathbf{K}_i}{j\omega - p_i} + \mathbf{K}_\infty + j\omega\mathbf{K}_0 \qquad (3)$$

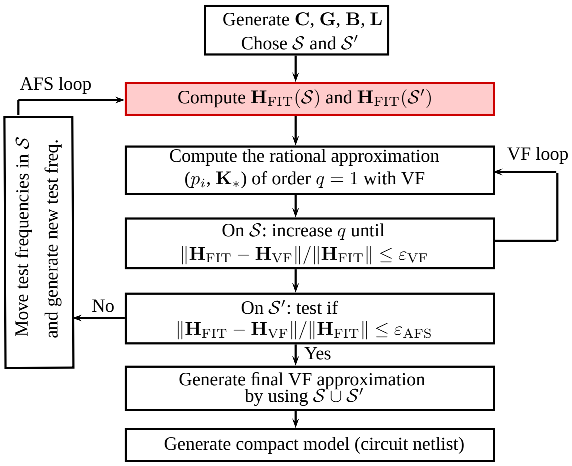

Fig. 1. VF based AFS algorithm for compact model extraction. The filled block runs in parallel: each frequency is solved on a node (distributed); each system is solved on a core (in parallel).

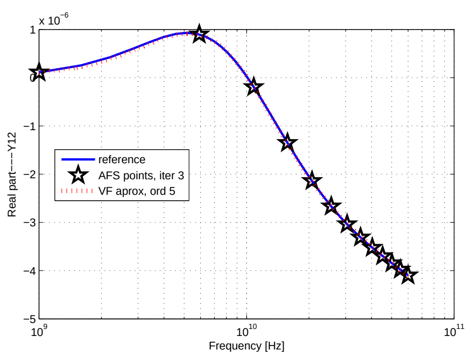

Fig. 2. Real benckmark - minimal FIT grid: 13 points were needed to extract a reduced model of order 5. The reference curve is simulated by oversampling.

for $\mathbf{H}_{\mathrm{FIT}}(\omega)$. Here q is the order of the reduced system, which can be automatically found. The problem to be solved is the following. Given the semi-state space matrices, and the frequency range, find a minimal set of sample frequencies $\mathcal{S} = \{\omega_k, k = 1, F\}$, distributed in the frequency range, so that the relative error $\|\mathbf{H}_{\mathrm{FIT}}(\omega_i') - \mathbf{H}_{\mathrm{FIT}}(\omega_i')\| / \|\mathbf{H}_{\mathrm{FIT}}(\omega_i')\|$ computed for every $\omega_i' \in \mathcal{S}'$ is smaller than an imposed threshold $\varepsilon_{\mathrm{AFS}}$, where $\mathcal{S}'$ is a set of test frequencies interleaved among the samples from $\mathcal{S}$. The algorithm we propose is shown in fig.1.

We studied the performance of the algorithm on a benchmarks consisting two coupled coils with a complicated geometry, so that the minimal discretization grid yielded a semi-state space system of 104102 degrees of freedom. The results are summarized in table I. By using AFS, only 13 simulation were enough to extract a reduced-order model with order $q = 5$ and an error under 0.01 (fig.2).

With 5 computing workers, the extraction process is 11 times faster than the serial extraction with one node and one thread. All tests described above were run on a cluster which has in each node two INTEL Xeon Nehalem quad-core CPUs at 2.66GHz, 8MB cache and 24GB RAM. For this

TABLE I
MODEL EXTRACTION TIME, WITH 8 THREADS/WORKER, 4 STARTING FREQUENCIES, $\varepsilon_{\mathrm{AFS}} = 10^{-3}$

No. workers	Solving time [s]	Speed-up
1	291.19	3.81
2	165.4	6.71
3	120.29	9.23
4	98.5	11.27
5	97.3	11.41
Serial execution	1110.37	1

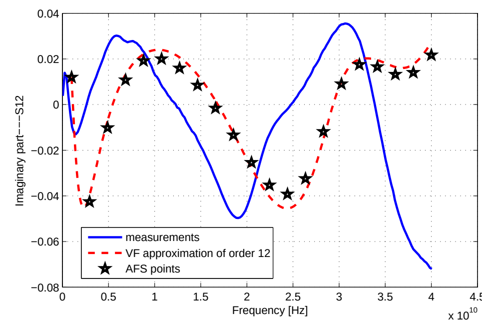

Fig. 3. Real benckmark - finer discretization: 21 points were needed to extract a reduced model of order 12. The reference curve is obtained from measurements.

test measurements are available thanks to *Austriamicrosystems* and preliminary simulation results are less than 5 % relative error in the scaterring parameters than the measurements. Fig.3 shows the comparison between measurements and the simulation results obtained with the proposed algorithm for a finer grid, corresponding to an initial model of order 210375. Only 21 points are needed to catch the characteristic landscape, meanwhile generating a reduced model of order 12. Our implementation use both distributed and shared memory programming models as well as industry-standard software tools and technologies that allowed an efficient cache and CPU use on individual nodes and minimal communication between nodes. The final paper will also discuss the difficulties encountered when the algorithm was run on a cluster that includes AMD nodes as well.

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